

A More Compact Expression of Relative Jacobian Based on Individual Manipulator Jacobians

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Abstract

This work presents a re-derivation of relative Jacobian matrix for parallel manipulators, expressed in terms of the individual manipulator Jacobians and multiplied by their corresponding transformation matrices. This is particularly useful when the individual manipulator Jacobians are given, such that one would not need to derive an entirely new expression of a relative Jacobian but will only use the existing manipulator Jacobians and perform the necessary transformations. In this work, the final result reveals a wrench transformation matrix which was not present in previous derivations, or was not explicitly expressed. The proposed Jacobian expression results in a simplified, more compact and intuitive form. It will be shown that the wrench transformation matrix is present in stationary as well as mobile combined manipulators. Simulation results show that at high angular end-effector velocities, the contribution of the wrench transformation matrix cannot be ignored.

Highlights: ▷ A relative Jacobian is derived and expressed in terms of the individual manipulator Jacobians. ▷ Previous forms of a relative Jacobian were unsimplified, and were not expressed in terms of the wrench transformation matrix. ▷ The simplified relative Jacobian presented in the work is more compact, intuitive, and is easier to implement when the manipulator Jacobians are given.

Keywords: Kinematics, combined manipulators, relative Jacobian, wrench transformation matrix

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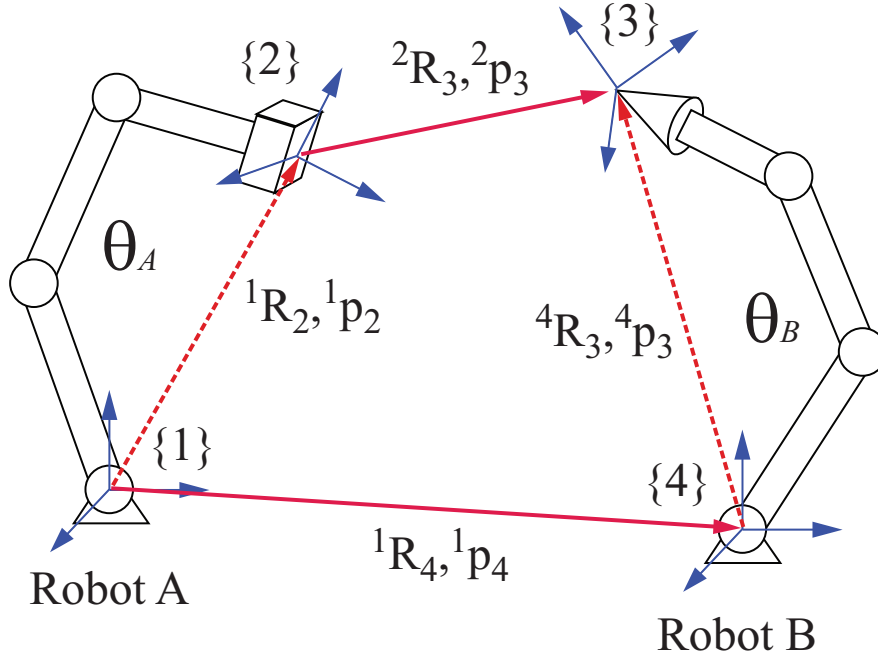


Figure 1: The dual-arm consisting for robot *A* and robot *B*, with attached frames {1}, {2}, {3}, and {4}. In this work, robot *A* is also called reference robot, while robot *B* is also called tool robot. The corresponding rotation matrices and position vectors with respect to the attached frames are shown.

1. Introduction

When manipulators are combined to perform a given task, the use of a relative Jacobian [1–4] affords a treatment of the combined manipulators as one single manipulator. The advantages of such an approach can be two-fold: (1) all the principles of single manipulator control can be applied to the combined manipulators, and (2) the dimension of the null space increased drastically compared to individually controlled manipulators. For example, given two six-degrees-of-freedom (6-DOFs) manipulators that move in the full space, no null space exists when the manipulators are controlled individually. But by treating the same manipulators as a single manipulator through the use of a relative Jacobian, a full-space motion allows a 6-DOFs null space [5]. Figure 1 shows a dual arm system with robot *A* (reference robot) and robot *B* (tool robot).

Pioneering work on relative Jacobian [1, 2] derived an entirely new formulation without using the existing Jacobians of the individual manipulators. Succeeding formulations were expressed in terms of the individual manipulator Jacobians [3, 4], but did not include the wrench transformation matrix [6–9]. More recent derivations [10–13] showed the in-

dividual manipulator Jacobians that are corrected in terms of the relative position vector rotations. However, the final expression remain in an unsimplified and non-compact form, and did not reveal the wrench transformation matrix. Furthermore, the case of mobile manipulators was not considered. Recent advances in combined manipulators that used the relative Jacobian approach include modular approach [14, 15], acceleration and torque redistribution [16], humanoid walking robot [17], behavioral control for autonomous systems [18], conflicting performance criteria [19], and cooperative manipulation [5, 20, 21]. Some advances on combined manipulators and dexterous manipulation include [22–26].

This work will present a step-by-step derivation of relative Jacobian for combined manipulators expressed in terms of the individual manipulator Jacobians, that results in a simplified and more compact form of the final expression. This is important because it makes the relative Jacobian expression more intuitive and easier to implement when the individual Jacobians are given.

Firstly, the proposed method is intuitive because it retains the information of each manipulator as a component of the single controller, such that the contribution of individual manipulator can be easily identified. As compared to more recent studies on relative Jacobian [10–13], where the individual manipulator Jacobians were not expressed independently. In addition, the necessary transformations for each individual Jacobian are explicitly given which give insight on understanding on the composition of the relative Jacobian.

Secondly, the proposed method can be easily implemented, when given the individual Jacobians, because it will not be necessary to derive an entirely new Jacobian of the combined manipulators. The method in this work will make use of the existing Jacobian of each manipulator, and will only require to derive the corresponding rotation and wrench transformation matrices. As compared to the earlier work on relative Jacobian [1, 2] which derived a totally new relative Jacobian, or where the results are unsimplified [10–13].

Thirdly the proposed method is more accurate at higher angular velocities of the reference robot end-effector, as compared to the case where the wrench transformation was not considered [3, 4]. Experimental results will be shown. In addition, an analytical approach will be presented to compare the proposed method against this previous formulation. Lastly this work will also present the relative Jacobian expression when the base of each manipulator is moving.

This work proceeds as follows. Section 2 shows the details of derivation for combined manipulators with stationary bases, while Section 2.2 shows the case when the bases are moving. Simulation results on the effect of the wrench transformation matrix at higher angular velocities is shown in Section 3. And lastly, the conclusion of this work is shown in Section 4.

2. Derivation of a More Compact Relative Jacobian

Figure 1 shows a dual-arm robot with its corresponding reference frames {1}, {2}, {3}, and {4} used in the derivation for a relative Jacobian. Appendix A presents preliminary derivations in rotational and translational velocities. The final form shows individual manipulator Jacobians with their corresponding transformation matrices.

2.1. The Case of Stationary Bases

With reference to Appendix A, the following were defined. A given dual-arm robot consists of two manipulators A and B (as shown in Fig. 1), with corresponding Jacobians $J_A = [J_{pA}; J_{oA}]$ and $J_B = [J_{pB}; J_{oB}]$ that are expressed in terms of the position and orientation velocity components. Its joint velocities are $\dot{\theta}_A$ and $\dot{\theta}_B$. Position ${}^i p_j$ and orientation ${}^i R_j$ terms are expressed with respect to frames i and j , and their corresponding velocities are ${}^i \dot{p}_j$ and ${}^i \omega_j$. Lastly, $S({}^i p_j)$ is a skew-symmetric matrix with input vector ${}^i p_j$, and I is an identity matrix.

From Appendix A, we combine the translational and rotational velocities together to get

$$\begin{aligned} \begin{bmatrix} {}^2 \dot{p}_3 \\ {}^2 \omega_3 \end{bmatrix} &= \begin{bmatrix} -{}^2 R_1 J_{pA} \dot{\theta}_A + S({}^2 p_3) {}^2 R_1 J_{oA} \dot{\theta}_A + {}^2 R_4 J_{pB} \dot{\theta}_B \\ -{}^2 R_1 J_{oA} \dot{\theta}_A + {}^2 R_4 J_{oB} \dot{\theta}_B \end{bmatrix} \\ &= \begin{bmatrix} -{}^2 R_1 J_{pA} + S({}^2 p_3) {}^2 R_1 J_{oA} & {}^2 R_4 J_{pB} \\ -{}^2 R_1 J_{oA} & {}^2 R_4 J_{oB} \end{bmatrix} \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix} \\ &= \begin{bmatrix} I & -S({}^2 p_3) \\ 0 & I \end{bmatrix} \begin{bmatrix} -{}^2 R_1 & 0 \\ 0 & -{}^2 R_1 \end{bmatrix} \begin{bmatrix} J_{pA} \\ J_{oA} \end{bmatrix} \begin{bmatrix} {}^2 R_4 & 0 \\ 0 & {}^2 R_4 \end{bmatrix} \begin{bmatrix} J_{pB} \\ J_{oB} \end{bmatrix} \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix} \end{aligned} \quad (1)$$

Thus a more compact form of the relative Jacobian, J_R , can be expressed as

$$J_R = \begin{bmatrix} -{}^2 \Psi_3 {}^2 \Omega_1 J_A & {}^2 \Omega_4 J_B \end{bmatrix}, \quad (2)$$

where

$${}^i \Psi_j = \begin{bmatrix} I & -S({}^i p_j) \\ 0 & I \end{bmatrix} \quad \text{and} \quad {}^i \Omega_j = \begin{bmatrix} {}^i R_j & 0 \\ 0 & {}^i R_j \end{bmatrix}. \quad (3)$$

The wrench transformation matrix, ${}^2 \Psi_3$, did not appear in the previous expressions of the relative Jacobian. This term is normally present in parallel mechanisms [6–8], which in this case, the dual-arm is in effect a parallel mechanism, where ${}^2 p_3$ is considered constant and rotating about the robot A end-effector. The contribution of this term in the relative translational velocity can be negligible when the rotational velocity of the robot A end-effector is close to zero.

As shown in (2), it is clearly seen that in order to derive the relative Jacobian J_R using the proposed method, one only needs to derive the wrench transformation matrix ${}^i\Psi_j$ and the rotation matrix ${}^i\Omega_j$, then incorporate the Jacobians of the standalone manipulators J_A and J_B to form the relative Jacobian J_R . In this way, when the individual manipulator Jacobians are given, there is no need to derive the relative Jacobian from scratch as shown in earlier studies [1, 2], or compute a number of unsimplified terms [10–13]. This supports the claim in this work for ease of implementation. In addition, because the components of the relative Jacobian are effectively modular, one or both robots can be removed and replaced other robots and their corresponding Jacobians. Or when locations of the robot bases changed, only the transformation matrices need to be modified.

The claim of intuitiveness of the proposed method is supported in the following. The standalone Jacobians are explicitly shown such that the contribution of each robot to the relative Jacobian J_R is clearly identifiable. Thus, the individual control for each manipulator can still be performed but it now becomes a component of the overall controller of the combined manipulators. As a result, each manipulator does not lose its individuality and, at the same time, the resulting controller enjoys the simplicity of a single manipulator control. In addition, the corresponding rotation matrix ${}^i\Omega_j$ transforms each manipulator Jacobian from its base frame to the global reference frame (end-effector of reference robot A). Furthermore, only J_A is multiplied by wrench transformation matrix ${}^i\Psi_j$ because its end-effector is where the moving global reference frame is attached. This is analogous to a parallel mechanism with wrench transformation matrices in its Jacobian [6–8], where the global reference frame is attached to the moving platform. Such insights on the contribution of the individual Jacobians and their corresponding transformations would not have been possible if the terms were not explicitly grouped.

2.2. The Case of Mobile Bases

In Appendix A, we assumed that the bases are not moving, that is, ${}^1\dot{R}_4 = {}^1\dot{p}_4 = {}^1\omega_4 = 0$. In this subsection, we will consider the case when the values of these terms are nonzero.

2.2.1. Rotation

From (A.1), we set ${}^1\dot{R}_4 \neq 0$ such that the middle term in its last expression becomes

$${}^2R_1 {}^1\dot{R}_4 {}^4R_3 = {}^2R_1 S({}^1\omega_4) {}^1R_4 {}^4R_3 = {}^2R_1 S({}^1\omega_4) {}^2R_1^T {}^2R_3 \quad (4)$$

where we repeat the same process as in (A.3), that is, we cancel 2R_3 and the $S(\omega)$ operator. And by using $RS(\omega)R^T = S(R\omega)$ [27], we get the final term for the moving base to be

$${}^2\omega'_3 = {}^2R_1 {}^1\omega_4 \quad (5)$$

which we will add later to (A.4) to combine the mobile base and manipulator terms. By simplifying further the above equation, we get

$$\begin{aligned}
{}^2\omega'_3 &= {}^2R_1({}^1\omega_0 + {}^1R_0^0\omega_4) \\
&= -{}^2R_1{}^1R_0^0\omega_1 + {}^2R_1{}^1R_0^0\omega_4 \\
&= -{}^2R_0^0\omega_1 + {}^2R_0^0\omega_4
\end{aligned} \tag{6}$$

2.2.2. Translation

By setting ${}^1\omega_4 \neq 0$ and ${}^1\dot{p}_4 \neq 0$, the cancelled terms in (A.7) to (A.8) becomes,

$$\begin{aligned}
{}^2\dot{p}'_3 &= -S({}^2R_4^4 p_3) {}^2R_1{}^1\omega_4 + {}^2R_1{}^1\dot{p}_4 \\
&= -S({}^2R_4^4 p_3) {}^2R_1(-{}^1R_0^0\omega_1 + {}^1R_0^0\omega_4) \\
&\quad + {}^2R_1(-{}^1R_0^0\dot{p}_1 + {}^1R_0^0\dot{p}_4) \\
&= S({}^2p_3) {}^2R_0^0\omega_1 - S({}^2p_3) {}^2R_0^0\omega_4 - {}^2R_0^0\dot{p}_1 + {}^2R_0^0\dot{p}_4
\end{aligned} \tag{7}$$

2.2.3. Combined Rotation and Translation

By combining the results in (6) and (7), we get,

$$\begin{bmatrix} {}^2\dot{p}'_3 \\ {}^2\omega'_3 \end{bmatrix} = \begin{bmatrix} -{}^2R_0^0\dot{p}_1 + S({}^2p_3) {}^2R_0^0\omega_1 + {}^2R_0^0\dot{p}_4 - S({}^2p_3) {}^2R_0^0\omega_4 \\ -{}^2R_0^0\omega_1 + {}^2R_0^0\omega_4 \end{bmatrix} \tag{8}$$

Then we set the motion of mobile base of robot A to be

$$\begin{bmatrix} {}^0\dot{p}_1 \\ {}^0\omega_1 \end{bmatrix} = \begin{bmatrix} J_{bpA}\dot{\theta}_{bA} \\ J_{boA}\dot{\theta}_{bA} \end{bmatrix} = J_{bA}\dot{\theta}_{bA} \tag{9}$$

where J_{bA} is the Jacobian for the base of robot A with corresponding position, J_{bpA} , and orientation, J_{boA} , parts and $\dot{\theta}_{bA}$ is the configuration space velocity of the robot A base. Whereas the motion of the base of robot B can be expressed as

$$\begin{bmatrix} {}^0\dot{p}_4 \\ {}^0\omega_4 \end{bmatrix} = \begin{bmatrix} J_{bpB}\dot{\theta}_{bB} \\ J_{boB}\dot{\theta}_{bB} \end{bmatrix} = J_{bB}\dot{\theta}_{bB} \tag{10}$$

where J_{bB} is the Jacobian for the base of robot B, and is composed of position, J_{bpB} , and orientation, J_{boB} , parts. The configuration space velocity of the robot B base is $\dot{\theta}_{bB}$.

So we substitute the Jacobian terms in (9) and (10) into (8) to get

$$\begin{aligned}
& \begin{bmatrix} -{}^2R_0 J_{bpA} + S({}^2p_3) {}^2R_0 J_{boA} & {}^2R_0 J_{bpB} - S({}^2p_3) {}^2R_0 J_{boB} \\ -{}^2R_0 J_{boA} & {}^2R_0 J_{boB} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{bA} \\ \dot{\theta}_{bB} \end{bmatrix} \\
&= \begin{bmatrix} I & -S({}^2p_3) \\ 0 & I \end{bmatrix} \begin{bmatrix} -{}^2R_0 & 0 \\ 0 & -{}^2R_0 \end{bmatrix} J_{bA} \begin{bmatrix} I & -S({}^2p_3) \\ 0 & I \end{bmatrix} \begin{bmatrix} {}^2R_0 & 0 \\ 0 & {}^2R_0 \end{bmatrix} J_{bB} \begin{bmatrix} \dot{\theta}_{bA} \\ \dot{\theta}_{bB} \end{bmatrix}. \tag{11}
\end{aligned}$$

By combining the manipulator and base velocities we get,

$$\begin{bmatrix} {}^2\dot{p}_{3T} \\ {}^2\omega_{3T} \end{bmatrix} = \begin{bmatrix} {}^2\dot{p}_3 + {}^2\dot{p}'_3 \\ {}^2\omega_3 + {}^2\omega'_3 \end{bmatrix} = J_{RT} \begin{bmatrix} \dot{\theta}_{bA} \\ \dot{\theta}_A \\ \dot{\theta}_{bB} \\ \dot{\theta}_B \end{bmatrix} \tag{12}$$

where J_{RT} is the total relative Jacobian of the combined manipulators and mobile bases, and is expressed as

$$J_{RT} = \begin{bmatrix} -{}^2\Psi_3 {}^2\Omega_0 J_{bA} & -{}^2\Psi_3 {}^2\Omega_1 J_A & {}^2\Psi_3 {}^2\Omega_0 J_{bB} & {}^2\Omega_4 J_B \end{bmatrix}. \tag{13}$$

3. The Effect of the Wrench Transformation Matrix

This paper builds on the expression of the wrench transformation matrix that results in a simplified, compact, and more intuitive expression of the relative Jacobian. It is further claimed that the inclusion of the wrench transformation matrix results in greater accuracy. The purpose of this section is to show such effect and compare it with the results when the wrench transformation matrix is not considered [3, 4]. Particularly, the results will show the contribution of $-S({}^2p_3)$ of the wrench transformation matrix in (1). Both analytical approach and numerical simulation are shown.

3.1. Analytical Approach

Let J'_R be the previous relative Jacobian without the transformation matrix [3, 4]. We want to prove that the statement “the null space of J_R is in the row space of J'_R ” is false. Let us assume that J_A and J_B are 6×6 and have full rank such that that J_R and J'_R are 6×12 and are full rank.

Suppose that the null space of J'_R lies in the row space of J_R . Since both of these linear spaces have the same dimension, and one is contained in the other, it follows that they are equal. We then have the row spaces of J'_R and J_R that are orthogonal to each other. This can be written as

$$J_R J'^T_R = 0. \tag{14}$$

From (2) (where we omit the Ω 's for simplicity), we have J'_R and J_R of the form

$$\begin{aligned} J'_R &= \begin{bmatrix} J_A & J_B \end{bmatrix} \\ J_R &= \begin{bmatrix} \begin{bmatrix} I & -S \\ 0 & I \end{bmatrix} J_A & J_B \end{bmatrix} \end{aligned} \quad (15)$$

Substituting the above expressions into (14) gives

$$\begin{aligned} 0 &= \begin{bmatrix} \begin{bmatrix} 0 & -S \\ 0 & 0 \end{bmatrix} J_A + J_A & J_B \end{bmatrix} \begin{bmatrix} J_A^T \\ J_B^T \end{bmatrix} \\ 0 &= \begin{bmatrix} 0 & -S \\ 0 & 0 \end{bmatrix} J_A J_A^T + J_A J_A^T + J_B J_B^T \\ \begin{bmatrix} 0 & S \\ 0 & 0 \end{bmatrix} J_A J_A^T &= J_A J_A^T + J_B J_B^T \end{aligned} \quad (16)$$

Since J_A is full rank and thus invertible, we have

$$\begin{bmatrix} 0 & S \\ 0 & 0 \end{bmatrix} = (J_A J_A^T + J_B J_B^T) (J_A J_A^T)^{-1} \quad (17)$$

Now the 6×6 matrices $J_A J_A^T$ and $J_B J_B^T$ are symmetric, positive definite so their sum $J_A J_A^T + J_B J_B^T$ is also symmetric, positive definite and hence has full rank. As the product of two square full rank matrices, the right hand side of (17) has full rank. However, the left hand side of (17) clearly does not have full rank (the last three rows are zero rows and the skew-symmetric matrix S has rank 2 or is the zero matrix). We thus conclude that (17) cannot hold, and consequently, (14) does not hold.

One implication in the above result is that there is a null vector of J_R that is not in the row space of J'_R . One possible interpretation to this is that there is a local self-motion that does not cause the robot with Jacobian J_R to move but will cause robot with Jacobian J'_R to move.

3.2. Numerical Simulation

The simulation consists of a primary task, that is the relative motion of the robot B end-effector with respect to the robot A end-effector, \dot{x}_R , and a secondary task, which is the motion of the robot A end-effector, \dot{x}_A , in the null space. The inverse kinematics solution is

$$\dot{q} = J_R^+ \dot{x}_R + (I - J_R^+ J_R) [J_A \ 0]^+ \dot{x}_A, \quad (18)$$

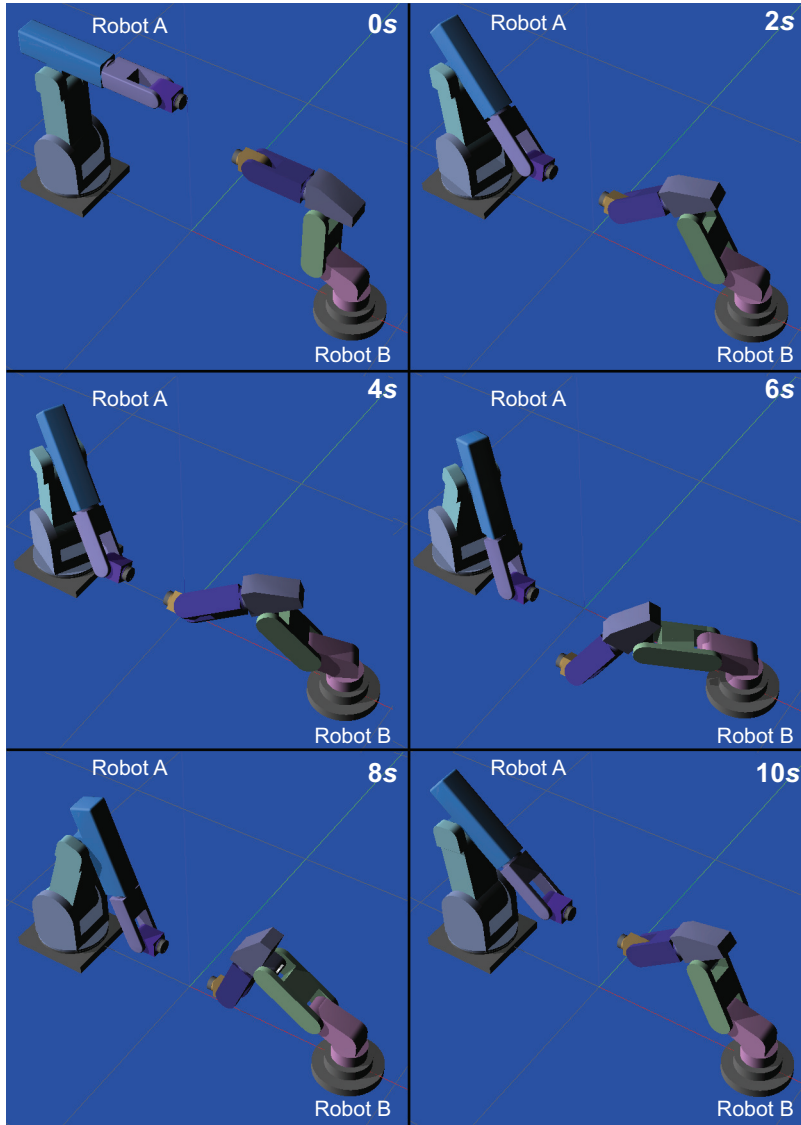


Figure 2: Snapshots of the dual arm taken at every two seconds where the robot A (reference robot) end-effector moves with respect to the robot B (tool robot) end-effector. Starting at 1s the reference robot end-effector draws a square while the tool robot end-effector draws a circle relative to the reference robot end-effector. The drawing task ended at 10s.

where \dot{x}_R draws a circle of radius 0.1 m and \dot{x}_A draws a square of side 0.2 m , while maintaining both end-effector orientations to be facing each other as shown in Fig. 2. Both manipulators start from rest, then goes to the starting location where the position of the robot A end-effector is $p_A = [0.5, 0, 0.4]^T$ and its end-effector orientation is facing forward. From rest to starting location, the robot B end-effector maintains a relative position $p_R = [0, 0, 0.24]^T$ and has a relative orientation that is facing the robot A end-effector. Once the starting location is achieved (at 1 s from rest), both end-effectors commence the assigned tasks simultaneously. For the same relative motion between end-effectors of robots A and B , the angular velocity of the reference robot (robot A) is varied by varying the velocity of its link six: (1) zero rotation, (2) one rotation per second, and (3) three rotations per second. The results are shown in Fig. 3.

With zero rotation of joint six of the reference robot, there is no change in the relative orientation between the end-effectors for the assigned motion. So as shown in Fig. 3a, the root-mean-square (RMS) errors with and without the wrench transformation matrix do not have significant difference. The maximum RMS error in both cases is around 0.1 mm . This can be a reason why in the previous work [3, 4], its contribution was ignored because it does not have much effect at low angular velocities of the reference robot end-effector.

As the angular velocity of the reference robot end-effector is increased (by increasing the joint six velocity to one rotation per second), a significant RMS error difference can be observed. Without considering the wrench transformation matrix, RMS error has a maximum value of 3.3 mm while the case with wrench transformation matrix has a maximum error of 0.2 mm . And lastly when the joint six velocity is further increased to three rotations per second, RMS error without the wrench transformation is around 10 cm while the case with wrench transformation matrix remains below half of a millimeter is around 0.45 mm .

4. Conclusion

This paper has shown a more compact expression of the relative Jacobian expressed in terms of the individual Jacobian matrices. This new expression revealed a wrench transformation matrix that was not present in previous derivations, or was not explicitly expressed. The proposed formulation is easier to implement, individual manipulator Jacobians are given, because one can use the existing manipulator Jacobians and will only need to derive the corresponding transformation matrices to arrive at the relative Jacobian. In addition, the method is modular such that when robots or base configurations changed, only the individual manipulator Jacobians or the transformation matrices change while the rest of the terms remain the same. The proposed formulation is more intuitive because the grouped expressions provided insight on the matrix operations that needed to be performed to the

RMS Position Error With and Without Wrench Transformation Matrix

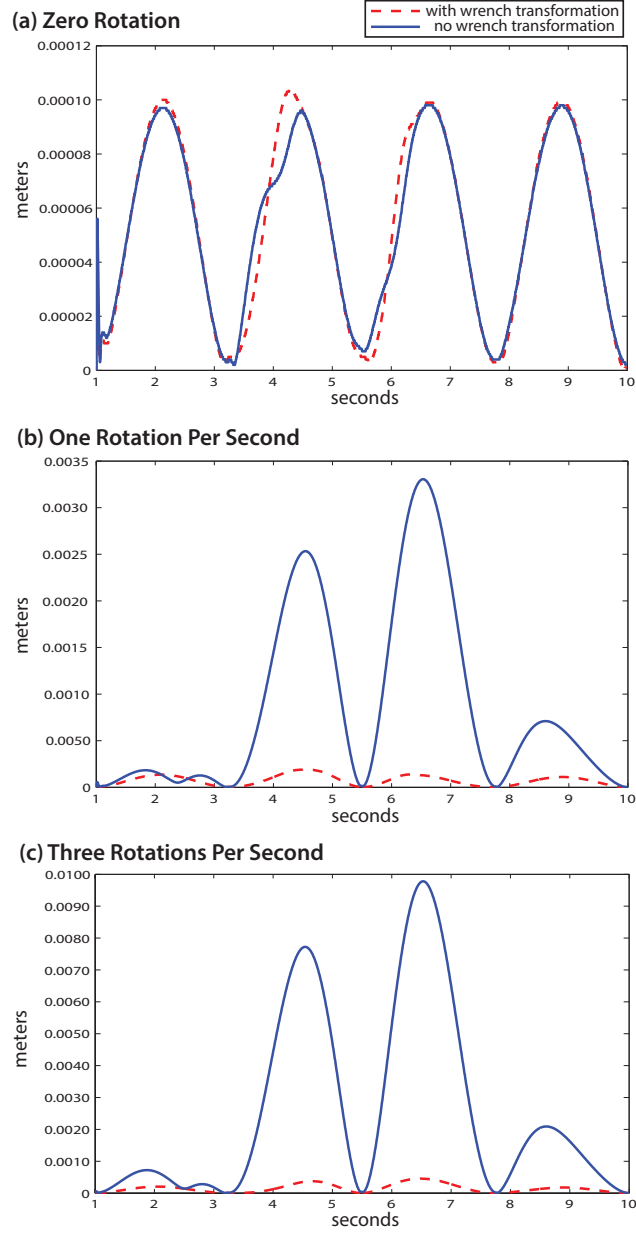


Figure 3: Root-mean-square (RMS) error of the relative position of the end-effector of robot *A* (tool robot) to the end-effector of robot *B* (reference robot). To verify the effect of the wrench transformation matrix at higher angular velocity of the reference robot end-effector, the joint six velocity of robot *B* is varied at: (a) zero rotation, (b) one rotation per second, and (c) three rotations per second. Results are compared with and without the wrench transformation matrix.

individual manipulator Jacobians in order to arrive at the relative Jacobian. Such insight would not been available if the terms were ungrouped and unsimplified. Analytical presentation and numerical simulations are used to compare the proposed approach against a previous formulation without the wrench transformation matrix.

Appendix A. A Preliminary Derivation for Relative Jacobian

The details of derivation for the simple rotation and translation velocities to be used in the relative Jacobian expression are shown in the following. The assigned reference frames {1}, {2}, {3}, and {4} are shown in Fig. 1.

Appendix A.1. Rotation

Let us start by deriving the angular velocities because it is simpler than the translational velocities. This approach will also show the skew-symmetric matrix simplification based on the method shown in [27]. The the assigned reference frames we can get the following rotational relationship,

$$\begin{aligned} {}^1R_2 {}^2R_3 &= {}^1R_4 {}^4R_3 \\ {}^2R_3 &= {}^2R_1 {}^1R_4 {}^4R_3 \\ {}^2\dot{R}_3 &= {}^2\dot{R}_1 {}^1R_4 {}^4R_3 + {}^2R_1 {}^1\dot{R}_4 {}^4R_3 + {}^2R_1 {}^1R_4 {}^4\dot{R}_3. \end{aligned} \tag{A.1}$$

When the bases are not rotating with respect to each other, ${}^1\dot{R}_4 = 0$. Note that $\dot{R} = \omega \times R = S(\omega)R$ [27], where ω is the corresponding angular velocity, such that $\omega = [\omega_x, \omega_y, \omega_z]^T$ and

$$S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{A.2}$$

is a skew-symmetric matrix used to replace the vector cross-product operation. When the bases are not rotating, (A.1) becomes

$$\begin{aligned} {}^2\dot{R}_3 &= {}^2\dot{R}_1 {}^1R_4 {}^4R_3 + {}^2R_1 {}^1R_4 {}^4\dot{R}_3 \\ S({}^2\omega_3) {}^2R_3 &= S({}^2\omega_1) {}^2R_1 {}^1R_4 {}^4R_3 + {}^2R_1 {}^1R_4 S({}^4\omega_3) {}^4R_3. \end{aligned} \tag{A.3}$$

And note that $RS(\omega)R^T = S(R\omega)$ as shown in [27]. Thus the previous equation becomes

$$\begin{aligned}
S({}^2\omega_3) {}^2R_3 &= S({}^2\omega_1) {}^2R_3 + {}^2R_4 S({}^4\omega_3) {}^2R_4^T {}^2R_4 {}^4R_3 \\
&= S({}^2\omega_1) {}^2R_3 + S({}^2R_4 {}^4\omega_3) {}^2R_3 \\
{}^2\omega_3 &= {}^2\omega_1 + {}^2R_4 {}^4\omega_3 \\
{}^2\omega_3 &= -{}^2R_1 {}^1\omega_2 + {}^2R_4 {}^4\omega_3
\end{aligned} \tag{A.4}$$

Given the Jacobian of robot A as J_A and the Jacobian of robot B as J_B , we separate the position and orientation component of the Jacobian of robot A to be $J_A = \begin{bmatrix} J_{pA} \\ J_{oA} \end{bmatrix}$, and of robot B to be $J_B = \begin{bmatrix} J_{pB} \\ J_{oB} \end{bmatrix}$, such that the above equation can be expressed as

$${}^2\omega_3 = -{}^2R_1 J_{oA} \dot{\theta}_A + {}^2R_4 J_{oB} \dot{\theta}_B. \tag{A.5}$$

Normally, robot A holds the workpiece and is called the reference robot, while robot B holds the tool and is called the tool robot.

Appendix A.2. Translation

Again, based on the assigned reference frames in Fig. 1, we can derive the following relationship,

$$\begin{aligned}
{}^1p_2 + {}^1R_2 {}^2p_3 &= {}^1p_4 + {}^1R_4 {}^4p_3 \\
{}^2p_3 &= {}^2R_1 ({}^1p_4 + {}^1R_4 {}^4p_3 - {}^1p_2) \\
&= {}^2R_1 {}^1p_4 + {}^2R_4 {}^4p_3 - {}^2R_1 {}^1p_2
\end{aligned} \tag{A.6}$$

Then by taking the time derivative,

$${}^2\dot{p}_3 = {}^2\dot{R}_1 {}^1p_4 + {}^2R_1 {}^1\dot{p}_4 + {}^2\dot{R}_4 {}^4p_3 + {}^2R_4 {}^4\dot{p}_3 - {}^2\dot{R}_1 {}^1p_2 - {}^2R_1 {}^1\dot{p}_2 \tag{A.7}$$

and because the bases are not moving, ${}^1\dot{p}_4 = 0$, which results in

$$\begin{aligned}
{}^2\dot{p}_3 &= S({}^2\omega_1) {}^2R_1 {}^1p_4 + S({}^2\omega_4) {}^2R_4 {}^4p_3 + {}^2R_4 {}^4\dot{p}_3 \\
&\quad - S({}^2\omega_1) {}^2R_1 {}^1p_2 - {}^2R_1 {}^1\dot{p}_2 \\
&= -S({}^2R_1 {}^1p_4) {}^2\omega_1 - S({}^2R_4 {}^4p_3) {}^2\omega_4 + {}^2R_4 {}^4\dot{p}_3 \\
&\quad + S({}^2R_1 {}^1p_2) {}^2\omega_1 - {}^2R_1 {}^1\dot{p}_2.
\end{aligned} \tag{A.8}$$

We note that ${}^2\omega_1 = -{}^2R_1^1\omega_2$ and ${}^2\omega_4 = {}^2\omega_1 + {}^2R_1^1\omega_4 = {}^2\omega_1$, because the bases are not moving such that ${}^1\omega_4 = 0$. This results in

$$\begin{aligned}
{}^2\dot{p}_3 &= S({}^2R_1^1p_4) {}^2R_1^1\omega_2 + S({}^2R_4^4p_3) {}^2R_1^1\omega_2 + {}^2R_4^4\dot{p}_3 \\
&\quad - S({}^2R_1^1p_2) {}^2R_1^1\omega_2 - {}^2R_1^1\dot{p}_2 \\
&= (S({}^2R_1^1p_4) + S({}^2R_4^4p_3) - S({}^2R_1^1p_2)) {}^2R_1^1\omega_2 + {}^2R_4^4\dot{p}_3 \\
&\quad - {}^2R_1^1\dot{p}_2 \\
&= S({}^2R_1^1p_4 + {}^2R_4^4p_3 - {}^2R_1^1p_2) {}^2R_1J_{oA}\dot{\theta}_A + {}^2R_4J_{pB}\dot{\theta}_B \\
&\quad - {}^2R_1J_{pA}\dot{\theta}_A.
\end{aligned} \tag{A.9}$$

Then we assign ${}^2p_3 = {}^2R_1^1p_4 + {}^2R_4^4p_3 - {}^2R_1^1p_2$ to get

$${}^2\dot{p}_3 = S({}^2p_3) {}^2R_1J_{oA}\dot{\theta}_A + {}^2R_4J_{pB}\dot{\theta}_B - {}^2R_1J_{pA}\dot{\theta}_A. \tag{A.10}$$

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